GUJARAT UNIVERSITY

B.E. SEM – 3 (Aero)

Question Bank

Maths – II

Each question is of equal Marks (10 Marks)

Q.1	Find the Fourier Series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.
Q.2	Expand $f(x)=x\sin x$ as a Fourier series in the interval $0<x<2\pi$.
Q.3	Find the Fourier series of $f(x)=2x-x^2$ in the interval (0,3). Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-...=\frac{\pi^2}{12}$.
Q.4	Find the Fourier series of the function $f(x)=\begin{cases} x^2 & 0\le x\le\pi \\ -x^2 & -\pi\le x\le 0\end{cases}$.
Q.5	Find the Fourier series of the function $f(x)=\begin{cases} \pi x & 0<x<1 \\ 0 & x=1 \\ \pi(x-2) & 1<x<2\end{cases}$. Hence show that $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$.
Q.6	Find the Fourier series of $f(x)=x^2$ in the interval $0<x<a$, $f(x+a)=f(x)$.
Q.7	If $f(x)=\lvert\cos x\rvert$, expand $f(x)$ as a Fourier series in the interval $(-\pi,\pi)$, $f(x+2\pi)=f(x)$.
Q.8	For the function $f(x)$ defined by $f(x)=\lvert x\rvert$, in the interval $(-\pi,\pi)$. Obtain the Fourier series. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...=\frac{\pi^2}{8}$.
Q.9	Given $f(x)=\begin{cases} -x+1 & -\pi\le x\le 0 \\ x+1 & 0\le x\le\pi\end{cases}$. Is the function even of odd ? Find the Fourier series for $f(x)$ and deduce the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....$
Q.10	Find the Fourier series of the periodic function $f(x)$; $f(x)=-k$ when $-\pi<x<0$ and $f(x)=k$ when $0<x<\pi$, and $f(x+2\pi)=f(x)$.
Q.11	Half range sine and cosine series of $f(x)=x(\pi-x)$ in $(0,\pi)$
Q.12	Find the Fourier series for the function $f(x)=\begin{cases} \pi x, 0<x<1 \\ \pi(x-2), 1<x<2\end{cases}$
Q.13	Find the Fourier series for f(x) defined by f(x) = $x+\frac{x^2}{4}$ when $-\pi< x <\pi$ and f(x + 2π) = f(x) and hence show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.......=\frac{\pi^2}{12}$

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Q.14	Find the Fourier series for the function $f(x)=\begin{cases} x; 0<x<1 \\ 0; 1<x<2 \end{cases}$.
Q.15	If f(x) = x in $0 < x < \frac{\pi}{2}$ $= \pi - x$ in $\frac{\pi}{2} < x < \frac{3\pi}{2}$ $= x - 2\pi$ in $\frac{3\pi}{2} < x < 2\pi$ Prove that f(x) = $\frac{4}{\pi}\left\{\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \right\}$
Q.16	If f(x) = $\frac{x}{l}$ when $0 < x < l$ $= \frac{2l-x}{l}$ when $l < x < 2l$ Prove that f(x) $\frac{1}{2} - \frac{4}{\pi^2}\left(\frac{1}{I^2}\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + ...\right)$
Q.17	When x lies between $\pm\pi$ and p is not an integer, prove that sin px = $\frac{2}{\pi}\sin p\pi\left(\frac{\sin x}{1^2-p^2} - \frac{2\sin 2x}{2^2-p^2} + \frac{3\sin 3x}{3^2-p^2} - ....\right)$
Q.18	Find the Fourier series for the function $f(x)=e^{ax}$ in $(-l,l)$
Q.19	Half range sine and cosine series of $f(x)=2x-1$ in $(0,1)$
Q.20	Half range sine and cosine series of x^2 in $(0,\pi)$
Q.21	Find Half range sine and cosine series for $f(x)=(x-1)^2$ in $(0,1)$
Q.22	Evaluate: $L\{\sin 2t\cos 3t\}$, $L\{e^{-3t}(\cos 4t+\sin 2t)\}$

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Q.23	Evaluate: $L\left\{\sin^2 2t\right\}$, $L\left\{e^{-2t}\cos 3t\right\}$
Q.24	Evaluate: $L\left\{\dfrac{\sin 2t-\sin 3t}{t}\right\}$, $L\left\{t\int_0^t e^{-4t}\sin 3t\,dt\right\}$
Q.25	Evaluate: $L^{-1}\left\{\log\left(\dfrac{s+1}{s-1}\right)\right\}$, $L^{-1}\left\{\dfrac{s^2+s+2}{s^5}\right\}$
Q.26	Evaluate: $L^{-1}\left\{\cot^{-1}\dfrac{s}{a}\right\}, L^{-1}\left\{\dfrac{s-1}{(s-1)^2+4}\right\}$
Q.27	Evaluate: $L^{-1}\left\{\log\left(\dfrac{s+2}{s+3}\right)\right\}$, $L^{-1}\left\{\dfrac{s+2}{\left(s^2+4s+5\right)^2}\right\}$
Q.28	Evaluate: $L^{-1}\left\{\dfrac{1+2s}{(s+2)^2\,(s-1)^2}\right\}, L^{-1}\left\{\dfrac{s^2+s+3}{s^6}\right\}$
Q.29	Evaluate: $L^{-1}\left\{\dfrac{(s+1)^2}{s^3}\right\}$, $L^{-1}\left\{\tan^{-1}\dfrac{s}{a}\right\}$
Q.30	Find the Laplace Transform of f(t), where (i) $f(t) = t$ if $0 < t < \dfrac{a}{2}$, $f(t+a) = f(t)$ $= a - t$ if $\dfrac{a}{2} < t < a$
Q.31	Find the Laplace transform of the function $f(t)=\begin{cases}\sin\omega t; 0<t<\dfrac{\pi}{\omega}\\ 0; \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega}\end{cases}$ $\qquad f(t)=f(t+\dfrac{2\pi}{\omega})$
Q.32	Use convolution theorem to find the Laplace Inverse Transform of

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	(i) $\dfrac{sa}{\left(s^2-a^2\right)^2}$ (ii) $\dfrac{s-2}{s\left(s-4s-13\right)}$
Q.33	Use convolution theorem to find the Laplace Inverse Transform of (i) $\dfrac{s^2}{\left(s^2+a^2\right)\left(s^2-b^2\right)}$ (ii) $\dfrac{1}{s^2\left(s-2\right)}$
Q.34	Find the value of the integral using Laplace Transform technique. (i) $\int_0^\infty t\, e^{-2t} \cos t\, dt$ (ii) $\int_0^t e^{-t} \dfrac{\sin t}{t}\, dt$
Q.35	Solve the initial value problem $y''+5y'+2y=e^{-2t}, y(0)=1, y'(0)=1,$ Using Laplace transformation.
Q.36	Solve the following Differential Equations using Laplace Transform technique. $\dfrac{d^2x}{dt^2}-2\dfrac{dx}{dt}+x=e^t$ *with* $x=2$ *and* $\dfrac{dx}{dt}=-1$ *at* $t=0$
Q.37	Solve the following Differential Equations using Laplace Transform technique. $\dfrac{d^2y}{dx^2}+y=1$ *with* $y(0)=1$ *and* $y\left[\dfrac{\pi}{2}\right]=0$
Q.38	Evaluate: $L^{-1}\left\{\dfrac{1+2s}{(s+2)(s-1)}e^{-3S}\right\}, L^{-1}\left\{\dfrac{(s+1)^2}{s^3}e^{-S}\right\}$
Q.39	Form the partial differential equation of following: (a) $2z=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$ (b) $z=f\left(x+ct\right)+g(x-ct)$
Q.40	Form the partial differential equation of following: (a) $2z=a^2x^2+b^2y^2$ (b) $z=x+y+f\left(xy\right)$
Q.41	Form the partial differential equation of following: (a) $z=\left(x^2+a\right)\left(y^2+b\right)$ (b) F(xy+z^2 , x + y+ z) = 0

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Q.42	Solve following partial differential equations : (a) $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ (b) $x(y-z)p + y(z-x)q = z(x-y)$
Q.43	Solve following partial differential equations : (a) $py + qx = pq$ (b) $z = px + qy + 2\sqrt{pq}$
Q.44	Solve following partial differential equations : (a) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y + xy$ (b) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$
Q.45	Solve following partial differential equations : (a) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+4y}$ (b) $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^3 + e^{x+2y}$
Q.46	(a) Solve: $\frac{\partial^2 z}{\partial x \partial y} = e^{-y}\cos x$,given that z = 0 when y = 0 and $\frac{\partial z}{\partial y} = 0$ when x = 0 (b) Solve: $\frac{\partial^2 z}{\partial x^2} = z$ given that $z = e^y$ and $\frac{\partial z}{\partial x} = e^{-y}$ when $x = 0$
Q.47	Solve: $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$ where z (x , 0) = 8 e^{-5x} using method of separation of variables.
Q.48	Solve: $3\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = 0$, where $z(x, 0) = 4e^{-x}$ by using method of separation of variables.
Q.49	Solve: $\frac{\partial z}{\partial x} = 4\frac{\partial z}{\partial y}$ where $z(0, y) = 8e^{-3y}$ using method of separation of variables.
Q.50	Attempt the following. 1) Express the function $f(x) = \begin{cases} 1; \lvert x \rvert < 1 \\ 0; \lvert x \rvert > 1 \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$. 2) Find the Fourier sine transform of $e^{-\lvert x \rvert}$.

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	Hence show that $\int_0^\infty \frac{x\sin mx}{i+x^2}dx=\frac{\pi e^{-m}}{2}$; $m>0$.
Q.51	Attempt the following. 1) Find the Fourier Transform of $f(x)=\begin{cases}1;\lvert x\rvert<1\\0;\lvert x\rvert>1\end{cases}$. Hence evaluate $\int_0^\infty \frac{\text{sinx}}{x}dx$. 2) Find the Fourier integral represent for $f(x)=\begin{cases}1-x^2;\lvert x\rvert\le 1\\0; \quad \lvert x\rvert>1\end{cases}$.
Q.52	Attempt the following. 1) Find the Fourier integral represent for $f(x)=\begin{cases}\mathrm{e}^{ax};\text{for } x\le 0, a>0\\\mathrm{e}^{-ax};\ for\ x\ge 0, a<0\end{cases}$. 2) Find the Fourier cosine transform of $f(x)=\begin{cases}1;0\le \mathrm{x}<2\\0; \quad x\ge 2\end{cases}$
Q.53	Attempt the following. 1) Using Fourier sine transform of $e^{-ax}(a>0)$, show that $\int_0^\infty \frac{x\sin kx}{a^2+x^2}dx=\frac{\pi e^{-ak}}{2}(k>0)$. 2) Using Fourier Integral show that $\int_0^\infty \frac{\omega\sin x\omega}{1+\omega^2}d\omega=\frac{\pi e^{-x}}{2}(x>0)$.
Q.54	Attempt the following. 1) Find the Fourier sine and cosine transform of $x^{n-1}\,(n>0)$. 2) Using Fourier integral representation, show that $\int_0^\infty \frac{\sin\omega\cos\omega x}{\omega}d\omega=\frac{\pi}{2}(0\le x<1)$